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Is Cosmic Censorship valid in Higher Dimensions?

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We study here the question if we can recover cosmic censorship by making a transition to higher dimensional spacetimes. It is pointed out that if only black holes are to result as end state of a continual gravitational collapse, several conditions must be imposed on the collapsing configuration, some of which would appear to be rather restrictive and we need to study carefully if these can be suitably motivated physically in a realistic collapse scenario. It would appear that in a generic higher dimensional collapse, both black holes and naked singularities would develop as end states of collapse.

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A considerable debate continues in recent years on the validity or otherwise of the Cosmic Censorship Conjecture (CCC) in black hole physics (see e.g. [1] for some recent reviews). The reason for this interest is that CCC is fundamental to many aspects of theory of black holes, and the astrophysical implications resulting from a continual gravitational collapse of a massive star, which has exhausted its nuclear fuel. As of today, no theoretical proof or any satisfactory mathematical formulation of CCC is available, where as many collapse scenarios have been analyzed where in the collapse end state is either a *black hole* (BH) or a *naked singularity* (NS), depending on the nature of the initial data from which the collapse evolves from a regular initial state to the final super-dense state. This has important astrophysical significance for the reason that naked singularities may have observational properties which are radically different from those of a black hole.

A note-worthy suggestion that has emerged towards a theoretical formulation of CCC is that any naked singularities resulting from matter models which may also develop singularities in special relativity, should not be regarded as physical (see e.g. Wald in [1]). Clearly, it will require a serious effort to cast this into a mathematical statement and proof for CCC. Also, it may not be easy to discard completely all the matter fields such as dust, perfect fluids, and matter with various other equations of state, which have been studied extensively in relativistic astrophysics for a long time.

Another possibility that indeed appears worth exploring is we may be actually living in a higher dimensional (HD) spacetime. The recent developments in string theory and other field theories strongly indicate that gravity is possibly a higher dimensional interaction, which reduces to the general relativistic description at lower energies. Hence, while CCC may fail in the 4-dimensional manifold of general relativity, it may well be restored due to the extra physical effects arising from our transition itself to a higher-dimensional spacetime. Such considerations should inspire a study of gravitational collapse in higher dimensions, and in fact in recent years many papers have reported results on spherically symmetric collapse of dust in HD [2]. It is obvious from the work so far on gravitational collapse that any possible proof of CCC must be inspired by such additional physical inputs into our current framework of thinking, and that any such alternatives are worth exploring due to the fundamental significance of CCC. The point is, if naked singularities did actually develop in realistic gravitational collapse of massive objects, they may have properties which would be rather different as compared to black holes.

From such a perspective, we investigate here the issue of BH/NS formations in a higher dimensional collapse in some detail, in order to bring out in a transparent manner the effect of dimensions on the final fate of the evolution of the matter cloud, which starts collapsing from a given regular initial data. A spherically symmetric collapse is considered in $N \geq 4$ dimensions and the matter form is taken to be dust as this is a well-studied case. Also, there have been various suggestions in the past that at the vicinity of the collapse, the in-falling velocity of the matter shells are so high that the effects of pressures are negligible and hence dust may be a good approximation. To predict the final state of collapse for a given initial mass and velocity profile, we study the singularity curve resulting from the collapsing shells, the tangent to which, at the singularity, is related to the radially outgoing null geodesic equation. Hence, by determining the sign of the tangent of the singularity curve at the central singularity we can definitely say, whether a future directed radially outgoing null geodesic comes out from the shell-focusing central singularity. It turns out that for CCC to be true *even in higher dimensions*, one must impose a number of constraints in the given

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system. The physical relevance of the same is discussed here.

To study the collapse of a spherically symmetric dust cloud, we choose a general spherically symmetric co-moving metric in $N \geq 4$ dimensions which has the form,

$$ds^2 = -dt^2 + e^{2\psi(t,r)}dr^2 + R^2(t,r)d\Omega^2_{N-2} \tag{1}$$

where,

$$d\Omega^2_{N-2} = \sum_{i=1}^{N-2}\left[\prod_{j=1}^{i-1}\sin^2(\theta^j)\right](d\theta^i)^2 \tag{2}$$

is the metric on $(N-2)$ sphere. The energy-momentum tensor of the dust has the form,

$$T^t_t = \rho(t,r);\ \ T^r_r = 0;\ \ T^{\theta^i}_{\theta^i} = 0 \tag{3}$$

We take the matter field to satisfy the *weak energy condition*, i.e. the energy density measured by any local observer be non-negative, and so for any timelike vector V^i, we must have,

$$T_{ik}V^iV^k \geq 0 \implies \rho \geq 0 \tag{4}$$

Using the above conditions, we can write the N-dimensional Einstein equations as [3],

$$(N-2)\left\{\frac{(N-3)}{2R^2}(1+\dot{R}^2 - R'^2e^{-2\psi}) + \frac{1}{R}(\dot{\psi}\dot{R} + e^{-2\psi}\psi'R' - e^{-2\psi}R'')\right\} = \rho \tag{5}$$

$$(N-2)\left\{\frac{N-3}{2R^2}(R'2e^{-2\psi} - \dot{R}^2 - 1) + \frac{\ddot{R}}{R}\right\} = 0 \tag{6}$$

$$\frac{(N-2)}{R}\left\{2(\dot{R}' - \dot{\psi}R')\right\} = 0\ \ ;\ \ {}^NG^i_i\ \ (i = 2,\cdots,N-2) = 0 \tag{7}$$

where ${}^NG^k_l$ is the N-dimensional Einstein tensor. Integrating the first part of equation (7) we get,

$$e^{2\psi} = \frac{R'^2}{1+f(r)} \tag{8}$$

where, $f(r)$ is an arbitrary function of co-ordinate r, and $f(r) > -1$. Hence the generalized Tolman-Bondi-Lemaitre (TBL) metric in N-dimensions becomes,

$$ds^2 = dt^2 - \frac{R'^2}{1+f(r)}dr^2 - R^2(t,r)d\Omega^2_{N-2} \tag{9}$$

Now substituting equation(8), in (5) and (6), we have,

$$\frac{(N-2)U'}{2R^{(N-2)}R'} = \rho,\quad \frac{(N-2)\dot{U}}{2R^{(N-2)}\dot{R}} = 0 \tag{10}$$

where we define and get by solving equation(10),

$$U = R^{(N-3)}(\dot{R}^2 - f(r))\ \ ;\ \ U = F(r) \tag{11}$$

Where $F(r)$ is another arbitrary function of co-ordinate r. In spherically symmetric spacetimes $F(r)$ is the *mass function*, which describes the mass distribution of the dust cloud and $f(r)$ is the *velocity function*, describing the velocity distribution of the collapsing shells. Thus using equations (10) and (11), we finally get the required *equations of motion* as,

$$\frac{(N-2)F'}{2R^{(N-2)}R'} = \rho\ \ ;\ \ \dot{R}^2 = \frac{F(r)}{R^{(N-3)}} + f(r) \tag{12}$$

The collapsing matter cloud condition implies that $\dot{R} < 0$. We use the scaling independence of co-ordinate r to write,

$$R(t,r) = rv(t,r) \quad ; \quad v(t_i,r) = 1 \quad ; \quad v(t_s(r),r) = 0 \quad ; \quad \dot{v} < 0 \tag{13}$$

This means we scale the co-ordinate r in such a way that at the initial epoch $R = r$ and at the singularity, $R = 0$. From the equations of motion it is evident that to have a regular solution over all space at the initial epoch, the two free functions $F(r)$ and $f(r)$ must have the following forms,

$$F(r) = r^{(N-1)}\mathcal{M}(r); \;\; f(r) = r^2 b_0(r) \tag{14}$$

Where $\mathcal{M}(r)$ and $b_0(r)$ are at least C^1 functions of r for $r = 0$, and at least a C^2 function for $r > 0$.

To focus now on the question of validity of CCC in a higher dimensional spacetime framework, consider a model initial density profile as given by,

$$\rho(t_i,r) = \rho_0 + r\rho_1 + r^2\frac{\rho_2}{2!} + r^3\frac{\rho_3}{3!} + \cdots \tag{15}$$

and we write the function $\mathcal{M}(r)$ as,

$$\mathcal{M}(r) = \sum_{n=0}^{\infty}\mathcal{M}_n r^n \quad ; \quad \mathcal{M}_n = \frac{2\rho_n}{(N-2)(N+n-1)n!} \tag{16}$$

With these regular initial conditions, equation(12) becomes,

$$v^{\frac{N-3}{2}}\dot{v} = -\sqrt{\mathcal{M}(r) + v^{(N-3)}b_0(r)} \tag{17}$$

Here the negative sign implies that $\dot{v} < 0$, *i.e.* the matter cloud is collapsing. Integrating the above equation with respect to v, we get,

$$t(v,r) = \int_v^1 \frac{v^{\frac{N-3}{2}}dv}{\sqrt{\mathcal{M}(r) + v^{(N-3)}b_0(r)}} \tag{18}$$

We note that the co-ordinate r is treated as a constant in the above equation. Expanding $t(v,r)$ around the center, we get,

$$t(v,r) = t(v,0) + r\mathcal{X}(v) + r^2\frac{\mathcal{X}_2(v)}{2} + r^3\frac{\mathcal{X}_3(v)}{6} + \cdots \tag{19}$$

where the function $\mathcal{X}(v)$ is given by,

$$\mathcal{X}(v) = -\frac{1}{2}\int_v^1 \frac{v^{\frac{N-3}{2}}(\mathcal{M}_1 + v^{(N-3)}b_1)dv}{(\mathcal{M}_0 + v^{(N-3)}b_{00})^{\frac{3}{2}}} \tag{20}$$

where,

$$b_{00} = b_0(0) \;\; ; \;\; \mathcal{M}_0 = \mathcal{M}(0) \;\; ; \;\; b_1 = b'(0) \;\; ; \;\; \mathcal{M}_1 = \mathcal{M}'(0) \tag{21}$$

Thus, the time taken for the central shell to reach the singularity is given as

$$t_{s_0} = \int_0^1 \frac{v^{\frac{N-3}{2}}dv}{\sqrt{\mathcal{M}_0 + v^{(N-3)}b_{00}}} \tag{22}$$

From the above equation it is clear that for t_{s_0} to be defined,

$$\mathcal{M}_0 + v^{(N-3)}b_{00} > 0 \tag{23}$$

Hence the time taken for other shells to reach the singularity can be given by the expansion,

$$t_s(r) = t_{s_0} + r\mathcal{X}(0) + \mathcal{O}(r^2) \tag{24}$$

Also, from equation(17) and (19) we get for small values of r, along constant v,

$$v^{\frac{N-3}{2}}v' = \sqrt{(\mathcal{M}_0 + v^{(N-3)}b_{00})}\,(\mathcal{X}(v) + r\mathcal{X}_2(v) + \cdots) \tag{25}$$

Now we can easily see that the value of $\mathcal{X}(0)$ depends on the functions b_0 and $\mathcal{M}$, which in turn depend on the initial data. Thus a given set of density and velocity profiles completely determines the tangent to the singularity curve at the singularity. Also it is evident that given any one of these two profiles we can always choose the other in such a way that the quantity $\mathcal{X}(0)$ will be either positive or negative.

The apparent horizon within the collapsing cloud is given by the equation,

$$\frac{F}{R^{N-3}} = 1 \tag{26}$$

which gives the boundary of the trapped surface region of the space-time. Broadly it can be stated that if the neighborhood of the center gets trapped earlier than the singularity, then it is covered, otherwise it is naked with families of non-spacelike future directed trajectories escaping from it.

In order to consider the possibility of existence of such families, and to examine the nature of the singularity occurring at $R = 0$, $r = 0$ for the scenario under consideration, let us consider the outgoing radial null geodesics equation,

$$\frac{dt}{dr} = e^{\psi} \tag{27}$$

The singularity occurs at a point $v(t_s(r), r) = 0$, which corresponds to $R(t_s(r), r) = 0$. Therefore, if we have any future directed null geodesics terminating in the past at the singularity, we must have $R \to 0$ as $t \to t_s$. Now writing equation(27) in terms of variables $(u = r^{\alpha}, R)$ where $\alpha > 1$, we have,

$$\frac{dR}{du} = \frac{1}{\alpha}r^{-(\alpha-1)}R'\left[1 + \frac{\dot{R}}{R'}e^{\psi}\right] \tag{28}$$

Choosing $\alpha = \frac{N+1}{N-1}$, and using equation(12) together with the collapse condition $\dot{R} < 0$, we get,

$$\frac{dR}{du} = \frac{N-1}{N+1}\left(\frac{R}{u} + \frac{v'v^{\frac{N-3}{2}}}{(\frac{R}{u})^{\frac{N-3}{2}}}\right)\left(\frac{1 - \frac{F}{R^{N-3}}}{e^{-\psi}R'(e^{-\psi}R' + \dot{R})}\right) \tag{29}$$

If null geodesics terminate at the singularity in the past with a definite tangent, then at the singularity the tangent to the geodesic $\frac{dR}{du} > 0$, in the (u, R) plane and must have a finite value. In our case, all singularities for $r > 0$ are covered since $\frac{F}{R} \to \infty$ and hence $\frac{dR}{du} \to -\infty$. Therefore only the singularity at the central shell could be naked. Let us define the tangent to the null geodesic at the singularity as,

$$x_0 = \lim_{t\to t_s}\lim_{r\to 0}\frac{R}{u} = \left.\frac{dR}{du}\right|_{t\to t_s; r\to 0} \tag{30}$$

Using equations (29) and (25), we get,

$$x_0^{\frac{N-1}{2}} = \frac{N-1}{2}\sqrt{\mathcal{M}_0}\mathcal{X}(0) \tag{31}$$

In the (t, r) plane, the null geodesic equation near the singularity will be,

$$t - t_s(0) = x_0 r^{\frac{N+1}{N-1}} \tag{32}$$

Now if $\mathcal{X}(0) > 0$, then $x_0 > 0$ and we get radially outgoing null geodesic from the singularity, making the singularity of the central shell a naked one. While if $\mathcal{X}(0) < 0$, we will have a black hole solution. If $\mathcal{X}(0) = 0$ then we will have to take into account the next higher order non-zero term in the singularity curve equation, and do a similar analysis by choosing a different value of α in equation (28).

It is now possible to examine the question of validity of cosmic censorship in a higher dimensional collapse scenario under consideration. Let us consider the class of *marginally bound* collapse for a more transparent understanding of the problem. This is the case when the velocity function $b_0(r)$ above vanishes identically for the collapsing shells. In this case, the coefficients $\mathcal{X}_n(0)$s as described in equation (19) are given by,

$$\mathcal{X}_n(0) = -\frac{n!}{N-1}\left(\frac{\mathcal{M}_n}{\mathcal{M}_0^{\frac{3}{2}}}\right) \tag{33}$$

The quantities $\mathcal{M}_n$ are described in equation (16). Now it is evident that whenever $\rho_1 < 0$, we will get a naked singularity *in all dimensions*, whereas $\rho_1 > 0$ always results in a black hole.

Let us, however, *assume* that the initial density distribution has odd terms in r vanishing, i.e. it admits no 'cusps' at the center and that it must be either sufficiently differentiable, or a smooth and analytic function of r. In that case, we must have $\rho_1 = 0$. Then we get from equation (25) that in the neighborhood of the singularity, the behavior of v is given by,

$$\lim_{t\to t_s}\lim_{r\to 0} v = \left[\frac{N-1}{4}\sqrt{\mathcal{M}_0}\mathcal{X}_2(0)\right]^{\frac{2}{N-1}} r^{\frac{4}{N-1}} \tag{34}$$

Also, at the same limit the function $\frac{F}{R^{N-3}}$ has the form

$$\lim_{t\to t_s}\lim_{r\to 0}\frac{F}{R^{N-3}} = \frac{r^2\mathcal{M}_0}{v^{(N-3)}} \tag{35}$$

Thus it is clear from equations (34) and (35), that if $N > 5$ then for $\lim_{t\to t_s}, \lim_{r\to 0}$, $\frac{F}{R} \to \infty$ and thus the end state of collapse will always be a black hole (see also Banerjee et al in [2]). *It thus follows that for a marginally bound dust collapse, with $\rho_1 = 0$, i.e when the initial density profile is sufficiently differentiable or smooth, the CCC is always respected in a higher dimensional spacetime with $N = 6$ or higher.*

It follows that if we can suitably motivate physically all the assumptions above, then we can restore CCC in higher dimensional spacetimes. Let us discuss each of these in some detail. That the equation of state must be dust-like in the final phases of collapse is a strong assumption, but it is not a possibility that can be completely ruled out (see e.g. [4]). After all, we know very little on the equations of state, especially how it would be like, in the advanced stages of collapse. Again, it is quite possible that in the very later stages of collapse the configuration is very much like a marginally bound one in the vicinity of the center.

All the same, in our view the assumption $\rho_1 = 0$ is a tricky one, and has been extensively discussed in the past. While it may be quite convenient to deal with smooth and analytic density profiles, especially when it comes to numerical models, it should not be forgotten that after all this is only an extra assumption, and that the basic equations of general relativity do not demand such a constraint. Neither it is clear astrophysically that the interiors of the stars must necessarily have smooth density profiles. In certain equilibrium cases, the field equations imply that they have to be smooth, but this need not be true in general, and especially the dynamically developing collapse situations could be very different. It should be also pointed out that observations have been noted suggesting the existence of cusps in globular cluster cores [5]. We thus conclude that each of the above assumptions require further scrutiny and sufficient physical motivation so as to arrive at any conclusion on the status of CCC in a higher dimensional spacetime.

It should be noted that an interesting critical behavior emerges as below in the case when we have $N = 5$. Again with the assumption $\rho_1 = 0$ for a five dimensional marginally bound case, using $\alpha = 2$ in equation (28), and using the condition for the naked singularity as $\mathcal{X}_2(0) \geq 0$ (and that the null geodesic coming out from the singularity must lie within the space time), we get a critical value for the quantity $\xi = \frac{M_2}{M_0^2}$ when ρ_2 is non-zero, for the transition from BH to NS phases. If $\xi \leq \xi_c = -8$ we would get a naked singularity, while for $\xi > \xi_c$ we will have a black hole. The scenario, for the case $N = 5$ is then let $\rho_1 \neq 0$, then collapse always ends in a naked singularity. Next, suppose $\rho_1 = 0$, however $\rho_2 \neq 0$. In that case when ρ_2 is beyond a certain critical value, we have black holes emerging out of collapse, but otherwise naked singularity results. This is similar to the usual critical value in four-dimensions obtained for ρ_3, for transition between the BH/NH phases, when $\rho_1 = \rho_2 = 0$ but $\rho_3 \neq 0$, and under similar assumptions as above [6].

Finally, let us consider the scenarios when some of the above assumptions break down. We have already noted above that when ρ_1 is non-vanishing, then the collapse ends in a naked singularity in all dimensions, including $N = 4$. Again, our considerations above immediately imply that whenever spacetime is *not* marginally bound, the collapse always ends in both the BH/NH phases developing in the above sense, it irrespective of ρ_1 being either zero or non-zero. That is, in a non-marginally bound case, even the condition $\rho_1 = 0$ does not save the CCC. Finally, consider the case if we must believe some how that in the later stages of collapse the form of matter cannot be dust-like, and that non-dust forms, and effects of pressures must be suitably taken into account. In such a situation, it is again known that even if one considered only Oppenheimer-Snyder like homogeneous initial density profiles (however, with non-zero pressures), even then the pressure by it-self can cause sufficient distortions in the formation of the apparent horizon, so as to again cause a naked singularity as end state of collapse, rather than a black hole [7].

Thus we conclude that there may be some hope, as outlined above, to recover CCC while we transit to a higher dimensional spacetime arena. This is subject to the validity of several extra physical inputs as we described above. On the other hand, once we move to more general situations of either a non-marginally bound case, or with a more general form of matter, or without any restrictive extra-assumptions on the nature of the density profiles, then generically both the BH/NS phases would result as end states of collapse even in a higher dimensional spacetime scenario. Further details on these aspects will be presented elsewhere.

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